MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020 SESSION

PTG 0116 – TRIGONOMETRY AND COORDINATE GEOMETRY

(All sections / Groups)

18 OCTOBER 2019
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 3 pages (excluding the cover page) with 4 questions and an appendix.
2. Answer all questions.
3. Unless stated otherwise, if an answer is given as a decimal, it should be rounded to **four** significant figures.
4. Write your answers in the Answer Booklet provided.
5. Show all relevant steps to obtain maximum marks.

Question 1

*A list of trigonometric identities are provided in the appendix.

(a) Perform the following without using the conversion function in a calculator. Show the relevant steps in your conversion.

(i) Convert 89.9841° to D°M'S'' form. Round your answer to the nearest second. [3 marks]

(ii) Convert 56°43'33'' to decimal degrees. Round your answer to three decimal places. [3 marks]

(iii) Convert 201.5° to radians. Round your answer to three decimal places. [2 marks]

(b) Let $\sin\beta = -\frac{1}{4}$ and $\tan\beta > 0$.

(i) State which quadrant β is in. [1 mark]

(ii) Use trigonometric identities to find the exact values of $\csc\beta$, $\cos\beta$, $\sec\beta$, $\tan\beta$ and $\cot\beta$. [5 marks]

(c) Establish the following identities. If you use any existing trigonometric identities in your proof, write them explicitly beside your working. [6 marks]

(i) $\tan\theta\cot\theta - \sin^2\theta = \cos^2\theta$

(ii) $\dfrac{1-\sin\theta}{\sec\theta} = \dfrac{\cos^3\theta}{1+\sin\theta}$

(d) Solve $\tan^2\left(2\theta + \frac{\pi}{2}\right) = 2$ for $0 \le \theta < \frac{\pi}{2}$. [5 marks]

Question 2

(a) Let $a = -2 - 4i$ and $b = -3 + 12i$. Find

(i) $-3\overline{a} + 5\overline{b}$, [3 marks]

(ii) $\dfrac{b}{a}$. [4 marks]

Express your answers in standard form.

(b) Convert the rectangular coordinates $(-\sqrt{3}, 2)$ to polar coordinates. Hence, write $-\sqrt{3} + 2i$ in polar form. [4 marks]

(c) Show that $\sin^2\theta + \cos\theta = 2$ in rectangular form is $3x^4 + y^4 + 3x^2y^2 = 0$. [3 marks]

(d) Given $A(3,2,10)$ and $B(-4,5,-1)$, find

(i) $\overrightarrow{BA}$, [2 marks]

(ii) the coordinates of the point whose position vector equals $\overrightarrow{BA}$. [1 mark]

Continued…

(e) Given $\underset{\sim}{a} = \langle -2,5,3 \rangle$ and $\underset{\sim}{b} = \langle 5,3,-2 \rangle$, find

(i) the unit vectors that are orthogonal to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$, [5 marks]

(ii) $|2\underset{\sim}{a} - \underset{\sim}{b}|$. [3 marks]

Question 3

(a) A circle is described by $x^2 + y^2 - 16x - 10y + 40 = 0$.

(i) Find the centre and radius of the circle. [4 marks]

(ii) Find the coordinates of the points on the circle with $x = 9$. [3 marks]

(iii) Using one of the points in (a)(ii), find an equation of the circle's diameter in slope-intercept form. [3 marks]

(b) An ellipse is described by the equation $2x^2 + 3y^2 + 68x + 60y + 872 = 0$.

(i) Give a definition of an ellipse in words. [1 mark]

(ii) Rearrange the equation such that it is in standard form. [4 marks]

(iii) Sketch the ellipse. Label the foci, vertices and centre, and give their coordinates. [7 marks]

(c) Determine whether the following lines are perpendicular. [3 marks]

$$2x + 5y - 25 = 0$$

$$5x + 2y - 10 = 0$$

Question 4

(a) Solve the following system of linear equations for x and y only. Use Cramer's rule.

$$x + 2y + 2z = -2$$

$$3z - 4y - x = -19$$

$$5y - 4z + 6x = 15$$

[13 marks]

(b) Find A^{-1} if $A = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 3 & 6 \\ -5 & 8 & 5 \end{bmatrix}$. [12 marks]

Continued...

APPENDIX

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin^2 A = \dfrac{1-\cos 2A}{2}$

$\cos^2 A = \dfrac{1+\cos 2A}{2}$

$\tan^2 A = \dfrac{1-\cos 2A}{1+\cos 2A}$

$\tan A = \dfrac{\sin 2A}{1+\cos 2A} = \dfrac{1-\cos 2A}{\sin 2A}$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$

$\sin A\cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$

$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\sin 2A = 2\sin A\cos A$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

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